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LETTER TO THE EDITOR

The constraint on potential and decomposition for (2+1)-dimensional integrable systems

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Abstract. Besides a sufficient condition, a necessary condition is given to determine some kind of consistent constraint on the potential of a (2+1)-dimensional integrable system, which cannot be obtained from the sufficient condition. Under this kind of constraint on the potential a (2+1)-dimensional integrable system can be decomposed into two commuting (1+1)-dimensional integrable systems, and can be further decomposed into three commuting finite-dimensional integrable Hamiltonian systems. The BKP hierarchy is taken as an illustrative example.

It was demonstrated (see, for example, [1–8]) that each equation in a hierarchy of (1+1)-dimensional integrable Hamiltonian systems can be decomposed into two commuting finite-dimensional integrable Hamiltonian systems by restricting the hierarchy of equations to some kind of finite-dimensional invariant submanifolds of their phase space. The main way to look for this kind of invariant submanifolds was proposed as follows [4–7]. Consider integrable Hamiltonian systems

$$u_{t_n} = K_n(u) = J\frac{\delta H_n}{\delta u} \tag{1}$$

where J is a Hamiltonian operator and $\delta/\delta u$ denotes the variational derivative. The associated auxiliary linear problems for (1) are supposed to be

$$L\psi = 0 \tag{2a}$$

$$\psi_{t_n} = A_n\psi. \tag{2b}$$

Let $F_i(u)$ be conserved densities of (1); it is shown [9] that the equation

$$\sum_{i=0}^{N}\frac{\delta F_i}{\delta u} = 0 \tag{3}$$

determines an invariant submanifold of the flow (1). As we already pointed out in the (1+1)-dimensional case [4–7], if we use (1.3) in the following way:

$$\frac{\delta F_0}{\delta u} + \sum_{j=1}^{N}\frac{\delta \lambda_j}{\delta u} = 0 \tag{4}$$

where λ is spectral parameter in (2), then the property of (3) guarantees that two commuting finite-dimensional integrable Hamiltonian systems can be obtained from

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(2*a*), (2*b*) and (3). We would like to point out that this method can be generalized to the (2+1)-dimensional case. Indeed, it is easy to see from (3) that the equation

$$J \sum_{i=0}^{N} \frac{\delta F_i}{\delta u} = 0$$

or equivalently

$$\sum_{i=0}^{N} \tilde{K}_i(u) = 0 \tag{5}$$

where $\tilde{K}_i(u)$ are symmetries of (1), also determines an invariant submanifold of (1). For example, $(\psi\psi^*)_x$ is a symmetry generator for the Kadomtsev–Petviashvili (KP) equation; here ψ and ψ^* are eigenfunctions of (2) associated with the DP equation and its adjoint version [10]. So for an arbitrary symmetry $\tilde{K}_0(u)$ of KP, (5) implies that

$$\tilde{K}_0(u) + \sum_{i=1}^{N} (\psi_i \psi_i^*)_x = 0 \tag{6}$$

determines an invariant submanifold of the KP equation. This property explains why (6) can be used to obtain consistent constraint on u to construct two commuting (1+1)-dimensional integrable Hamiltonian systems from (6) and (2) associated with the KP equation in [11, 12].

However, we also would like to emphasize that equation (or formula like (6)) is a sufficient condition for determining a consistent constraint on u. Indeed there are some other kinds of consistent constraints on u which cannot be obtained from the sufficient condition (5). These consistent constraints also enable us to decompose a (2+1)-dimensional integrable system into two commuting (1+1)-dimensional integrable systems and to obtain some kinds of solutions to the (2+1)-dimensional integrable systems through solving the two commuting (1+1)-dimensional integrable systems. For example, under the constraint $u = \psi_x$, (2*a*) and (2*b*) associated with the KP equation become the first and second equations respectively, in the Burgers hierarchy [12]. We want to find this kind of constraint from some necessary condition by directly using the conserved densities of (1).

Furthermore, combining the results in (1+1)- and (2+1)-dimensional cases, we want to point out generally that a (2+1)-dimensional integrable system can be decomposed into three commuting finite-dimensional integrable Hamiltonian systems, and the solution to the latter three commuting systems solves the former (2+1)-dimensional system. This also provides a way to obtain some kinds of solutions to (2+1)-dimensional integrable systems through solving three commuting finite-dimensional integrable Hamiltonian systems.

In the present letter the BKP hierarchy [13] will be considered as a model example. We have not found the symmetry generator in terms of ψ and ψ^* for the BKP equation yet. So we cannot use (5) to obtain the constraint on u. We will show how to use some necessary condition connected with the conserved densities of the BKP equation to find the consistent constraint on u, which makes the decomposition possible. Then we will present two kinds of decompositions of equation in the BKP hierarchy into three commuting finite-dimensional integrable Hamiltonian systems.

Consider the BKP hierarchy [13], the first equation in the hierarchy BKP_1 (the (2+1)-dimensional CDGKS equation) reads

$$u_{t_5} + \tfrac{1}{9}\partial_x^5 u + \tfrac{5}{9}u_{xxt_3} + \tfrac{5}{3}uu_{xxx} + \tfrac{5}{3}u_x u_{xx} - \tfrac{5}{3}uu_{t_3} + 5u^2 u_x - \tfrac{5}{3}u_x \partial_x^{-1} u_{t_3} - \tfrac{5}{9}\partial_x^{-1} u_{t_3 t_3} = 0 \tag{7}$$

which is obtained as the compatibility condition of the following linear problems [10]:

$$\psi_{t_3} = \psi_{xxx} + 3u\psi_x \tag{8a}$$

$$\psi_{t_5} = [\partial_x^5 + 5u\partial_x^3 + 5u_x\partial_x^2 + (\tfrac{5}{3}\partial_x^{-1}u_{t_3} + \tfrac{10}{3}u_{xx} + 5u^2)\partial]\psi. \tag{8b}$$

We now try to find a constraint on u like

$$u = f(\psi, \psi_x, \ldots) \tag{9}$$

where $f(\psi, \psi_x, \ldots)$ is a polynomial of ψ, $\psi_x, \ldots$, so that (8a) and (8b) under (9) become two commuting $(1+1)$-dimensional integrable Hamiltonian systems. Observe that if $F_i(u)$ are the conserved densities of (7) (see [10]), then $F_i(f(\psi, \psi_x, \ldots))$ must be the conserved densities of the following equation obtained from (8a)

$$\psi_{t_3} = \psi_{xxx} + f(\psi, \psi_x, \ldots)\psi_x. \tag{10}$$

This is the necessary condition for (9) to be consistent constraint on u. For the first conserved density $F_1(u) = u$, the necessary condition requires that $u = f(\psi, \psi_x, \ldots)$ satisfy the formula of conservation law

$$\frac{\partial f(\psi, \psi_x, \ldots)}{\partial t_3} = \frac{\partial g(\psi, \psi_x, \ldots)}{\partial x} \tag{11}$$

where $g(\psi, \psi_x, \ldots)$ is also a polynomial of ψ, $\psi_x, \ldots$. Notice the term $\partial_x^{-1}u_{t_3}$ appearing in (8b); the requirement that (8b) under (8a) and (9) be a pure differential equation also imposes (11) on f. So (11) is the first necessary condition that f must satisfy. To illustrate the idea, we first consider

$$u = f(\psi)$$

then (10) and (11) give

$$(\psi_{xxx} + f\psi_x)f_\psi = g_\psi\psi_x + g_{\psi_x}\psi_{xx} + g_{\psi_x}\psi_{xx} + g_{\psi_{xx}}\psi_{xxx}.$$

Comparing the coefficients of ψ_{xxx} leads to

$$g = f_\psi\psi_{xx} + g_1(\psi, \psi_x)$$

which together with the remaining terms gives

$$ff_\psi\psi_x = (f_{\psi\psi}\psi_{xx} + g_{1\psi})\psi_x + g_{1\psi_x}\psi_{xx}.$$

Similarly, we find from the coefficients of ψ_{xx} that

$$g_1 = -f_{\psi\psi}\psi_x + g_2(\psi)$$

and we get

$$ff_\psi\psi_x = -f_{\psi\psi\psi}\psi_x^2 + g_{2\psi}\psi_x$$

which immediately yields $f_{\psi\psi\psi} = 0$. So we find

$$f(\psi) = \alpha\psi + \beta\psi^2. \tag{12}$$

If we consider $u = f(\psi, \psi_x)$, in the exactly same way, we find that $f(\psi, \psi_x)$ has to satisfy either (12) or

$$f(\psi, \psi_x) = \alpha(\psi^k)_x. \tag{13}$$

However, for the second conserved density of BKP $F_2(u)=\partial_x^{-1}u_{t_3}$, it is easy to verify that $F_2(\alpha(\psi^k)_x)=\alpha(\psi^k)_{t_3}=\alpha k\psi^{k-1}(\psi_{xxx}+\alpha k\psi^{k-1}\psi_x^2)$ is not a conserved density for the equation (10) under (13). Thus the necessary condition excludes the choice (13). Indeed for the general form (9), in similar way we still find that there is only one choice for f given by (12). In the following we will show that $u=2\psi$ and $u=2\psi^2$ are consistent constraints on u.

Let

$$u=2\psi \tag{14}$$

then ($8a$) becomes the KdV equation [14]

$$\psi_{t_3}=\psi_{xxx}+6\psi\psi_x. \tag{15}$$

Using (14) and (15), we have

$$\partial_x^{-1}u_{t_3}=2\psi_{xx}+6\psi^2 \tag{16}$$

and it is then easy to varify that ($8b$) is transformed to

$$\psi_{t_5}=\partial_x^5\psi+10\psi\psi_{xxx}+20\psi_x\psi_{xx}+30\psi^2\psi_x \tag{17}$$

which is just the second equation KdV_5 in the KdV hierarchy. It is obvious that if ψ satisfies both commuting integrable systems (15) and (17), then $u=2\psi$ is a solution to (7).

If we set

$$u=2\psi^2 \tag{18}$$

then ($8a$) becomes the MKdV_3 equation [15]

$$\psi_{t_3}=6\psi^2\psi_x+\psi_{xxx}. \tag{19}$$

Notice from (18) and (19) that

$$\partial_x^{-1}u_{t_3}=6\psi^4+4\psi\psi_{xx}-2\psi_x^2. \tag{20}$$

A direct calculation then shows that ($8b$) is transformed to the second equation MKdV_5 in the MKdV hierarchy,

$$\psi_{t_5}=\partial_x^5\psi+10\psi^2\psi_{xxx}+40\psi\psi_x\psi_{xx}+30\psi^4\psi_x+10\psi_x^3. \tag{21}$$

Also, it is easy to see that if ψ solves both commuting integrable systems (19) and (21), then $u=2\psi^2$ satisfies (7).

Remark 1. The above results provide a way to obtain some kinds of solutions to the BKP_I equation through (14) or (18) by solving two commuting $(1+1)$-dimensional integrable systems (15) and (17) or (19) and (21), respectively.

Remark 2. By using (7) and (8), a direct calculation shows that $(\psi^2)_x$ does not satisfy the linearized BKP_I equation. This means that $(\psi^2)_x$ is not a symmetry of (7). So the constraint (18) cannot be obtained from the sufficient condition (5).

Remark 3. From the conserved densities of the BKP equation $F_i(u)$ [10], we can construct the conserved densities $\mu_i(\psi)$ for the KdV hierarchy and $\sigma_i(\psi)$ for the MKdV hierarchy by substituting (14), (15), and (18), (19), respectively, into $F_i(u)$. For example, from $F_1(u)=u$, $F_2(u)=\partial_x^{-1}u_{t_3}$, $F_3(u)=u\partial_x^{-1}u_{t_3}+\frac{1}{3}\partial_x^{-2}u_{t_3t_3}+u_x^2-u^3$, we have

$$\mu_1(\psi)=\psi \qquad \mu_2(\psi)=\psi_{xx}+3\psi^2 \qquad \mu_3(\psi)=2\psi^3-\psi_x^2$$

$$\sigma_1(\psi)=\psi^2 \qquad \sigma_2(\psi)=2(\psi\psi_x)_x-3\psi_x^2+3\psi_x^2+3\psi^4$$

$$\sigma_3(\psi)=6\psi^6-6\psi^2\psi_x^2+3\psi_{xx}^2+8\psi^3\psi_{xx}.$$

Indeed similar results hold for the whole BKP hierarchy. for example, the second equation BKP_2 in the BKP hierarchy is

$$\begin{aligned} u_{t_7}&+\tfrac{1}{27}\partial_x^7u+\tfrac{14}{9}u_x\partial_x^4u+\tfrac{7}{3}u_{xx}u_{xxx}+\tfrac{119}{3}u^2u_{xxx}+119uu_xu_{xx}+\tfrac{7}{3}u_x^3-\tfrac{7}{9}u_xu_{xt_3}-\tfrac{7}{9}u_{xt_3t_3}-\tfrac{77}{3}u^3u_x\\ &+\tfrac{28}{3}u^2u_{t_3}-\tfrac{49}{3}uu_{xxt_3}+7uu_x\partial_x^{-1}u_{t_3}-\tfrac{7}{3}u\partial_x^{-1}u_{t_3t_3}-\tfrac{7}{3}u_{t_3}\partial^{-1}u_{t_3}\\ &-\tfrac{7}{9}u_x\partial_x^{-2}u_{t_3t_3}-\tfrac{7}{27}\partial_x^{-2}t_{t_3t_3t_3}. \end{aligned} \tag{22}$$

The associated auxiliary linear problems can be constructed out by following the lines of [10]:

$$\psi_{t_3}=\psi_{xxx}+3u\psi_x \tag{23a}$$

$$\begin{aligned}\psi_{t_7}=[&\partial_x^7+7u\partial_x^5+14u_x\partial_x^4+(\tfrac{56}{3}u_{xx}+14u^2+\tfrac{7}{3}\partial_x^{-1}u_{t_3})\partial_x^3+(\tfrac{35}{3})u_{xxx}+28uu_x+\tfrac{7}{3}u_{t_3})\partial_x^2\\ &+(\tfrac{28}{9}\partial_x^4u+14uu_{xx}+7u_x^2+\tfrac{28}{9}u_{xt_3}+\tfrac{14}{3}u^3+7u\partial_x^{-1}u_{t_3}+\tfrac{7}{9}\partial_x^{-2}u_{t_3t_3})\partial_x]\psi.\end{aligned} \tag{23b}$$

A straightforward calculation shows that under the constraint $u=2\psi$ and (15), (23b) becomes the third equation KdV_7 in the KdV hierarchy [14]

$$\begin{aligned}\psi_{t_7}=\partial_x^7\psi&+42\psi_x\partial_x^4\psi+70\psi_{xx}\psi_{xxx}+14\psi\partial_x^5\psi+70\psi_x^3\\ &+280\psi\psi_x\psi_{xx}+70\psi^2\psi_{xxx}+140\psi^3\psi_x.\end{aligned} \tag{24}$$

This implies that if ψ satisfies both commuting integrable systems (15) and (24), then $u=2\psi$ solves (22).

Using the constraint $u=2\psi^2$ and (19), it is found from direct calculation that (23b) is transformed into the third equation MKdV_7 in the MKdV hierarchy [15]

$$\begin{aligned}\psi_{t_7}=\partial_x^7\psi&+14\psi^2\partial_x^5\psi+84\psi\psi_x\partial_x^4\psi+140\psi\psi_{xx}\psi_{xxx}+126\psi_x^2\psi_{xxx}+70\psi^4\psi_{xxx}+182\psi_x\psi_{xx}^2\\ &+560\psi^3\psi_x\psi_{xx}+420\psi^2\psi_x^3+140\psi^6\psi_x.\end{aligned} \tag{25}$$

Similarly, if ψ is a solution to both commuting integrable systems (19) and (25), then $u=2\psi^2$ satisfies (22).

It was shown in [5, 6] that each equation in the KdV hierarchy or MKdV hierarchy can be decomposed into two commuting finite-dimensional integrable Hamiltonian systems. For example, the KdV_3 equation (15) is associated with following auxiliary linear problems [14]:

$$\phi_{xx}+\psi\phi=\lambda\phi \tag{26a}$$

$$\phi_{t_3}=-\psi_x\phi+(4\lambda+2\psi)\phi_x. \tag{26b}$$

Notice that $\delta\lambda/\delta\psi=\phi^2$; we can obtain a consistent constraint on ψ from (4):

$$\psi=\sum_{j=1}^{N}\frac{\delta\lambda_j}{\delta\psi}=\sum_{j=1}^{N}\phi_j^2. \tag{27}$$

The property of (4) guarantees that (26*a*), (26*b*) and (27) are consistent. Indeed we can obtain two commuting finite-dimensional integrable Hamiltonian systems from (26*a*), (26*b*) and (27) for distinct λ_j [6]:

$$q_x = \frac{\partial H_0}{\partial p} \qquad p_x = -\frac{\partial H_0}{\partial q} \tag{28}$$

and

$$q_{t_3} = \frac{\partial H_3}{\partial p} \qquad p_{t_3} = -\frac{\partial H_3}{\partial q} \tag{29}$$

with

$$H_0 = F \equiv \tfrac{1}{2}\langle p, p\rangle - \tfrac{1}{2}\langle \Lambda q, q\rangle + \tfrac{1}{4}\langle q, q\rangle^2$$
$$H_3 = 4F_3$$

where $q = (q_1, \ldots, q_N)^{\mathrm{T}} \equiv (\phi_1, \ldots, \phi_N)^{\mathrm{T}}$, $p = (p_1, \ldots, p_N)^{\mathrm{T}} = (\phi_{1x}, \ldots, \phi_{Nx})^{\mathrm{T}}$, $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$, $\langle \cdot, \cdot \rangle$ denotes the inner product in R^N, F_k are involutive integrals of motion for (28) and (29) defined as follows [6]:

$$F_1 = 0$$

$$F_{k+1} = \tfrac{1}{2}[\langle \Lambda^{k-1} p, p\rangle - \langle \Lambda^k q, q\rangle + \tfrac{1}{2}\langle q, q\rangle\langle \Lambda^{k-1} q, q\rangle] + \tfrac{1}{4}\sum_{j=0}^{k-2} (\langle \Lambda^j p, p\rangle\langle \Lambda^{k-2-j} q, q\rangle$$
$$- \langle \Lambda^j p, q\rangle\langle \Lambda^{k-2-j} p, q\rangle) \qquad k = 1, 2, \ldots. \tag{30}$$

It is known [6] that if (p, q) is a solution of (28) and (29), then $\psi = \langle q, q\rangle$ satisfies the KdV$_3$ equation (15). Similarly it is shown in [6] that if (p, q) satisfies both commuting integrable Hamiltonian systems (28) and

$$q_{t_5} = \frac{\partial H_5}{\partial p} \qquad p_{t_5} = -\frac{\partial H_5}{\partial q} \tag{31}$$

with

$$H_5 = 4(2F_4 + F_2^2)$$

then $\psi = \langle q, q\rangle$ is a solution to KdV$_5$ equation (17). The above results imply that if (p, q) satisfies three commuting finite-dimensional integrable Hamiltonian systems (28), (29) and (31), then $u = 2\langle q, q\rangle$ is a solution to BKP$_1$ equation (7).

In the same way, we find that if (p, q) is a solution to three commuting finite-dimensional integrable Hamiltonian systems (28), (29) and

$$q_{t_7} = \frac{\partial H_7}{\partial p} \qquad p_{t_7} = -\frac{\partial H_7}{\partial q} \tag{32}$$

with (see [6])

$$H_7 = 32(F_5 + F_2 F_3)$$

then $u = 2\langle q, q\rangle$ satisfies the BKP$_2$ equation (22).

The auxiliary linear problems for MKdV$_3$ equation (19) are [15]:

$$\phi_x = \begin{pmatrix} \lambda & \psi \\ -\psi & -\lambda \end{pmatrix}\phi \qquad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \tag{33a}$$

$$\phi_{t_3} = \begin{pmatrix} 4\lambda^3 + 2\psi^2\lambda & 4\psi\lambda^2 + 2\psi_x\lambda + \psi_{xx} + 2\psi^3 \\ -4\psi\lambda^2 + 2\psi_x\lambda - \psi_{xx} - 2\psi^3 & -4\lambda^3 - 2\psi^2\lambda \end{pmatrix}\phi. \tag{33b}$$

Using the method in [5], we find that under the constraint on ψ:

$$\psi = \langle \Phi_1, \Phi_1 \rangle + \langle \Phi_2, \Phi_2 \rangle \tag{34}$$

where $\Phi_i = (\phi_{i1}, \ldots, \phi_{iN})$, we can obtain two commuting finite-dimensional integrable Hamiltonian systems from (33a) and (33b) for distinct λ_j:

$$\Phi_{1x} = \frac{\partial H_0}{\partial \Phi_2} \qquad \Phi_{2x} = -\frac{\partial H_0}{\partial \Phi_1} \tag{35}$$

$$\Phi_{1t_3} = \frac{\partial H_3}{\partial \Phi_2} \qquad \Phi_{2t_3} = -\frac{\partial H_3}{\partial \Phi_1} \tag{36}$$

with

$$H_0 = F_1 \equiv \langle \Lambda\Phi_1, \Phi_2 \rangle + \tfrac{1}{2}\langle \Phi_1, \Phi_1 \rangle \langle \Phi_2, \Phi_2 \rangle + \tfrac{1}{4}\langle \Phi_1, \Phi_1 \rangle^2 + \tfrac{1}{4}\langle \Phi_2, \Phi_2 \rangle^2$$

$$H_3 = 4F_2 + 4F_1^2$$

where F_k are the involutive integrals of motion for (35) and (36) defined as follows:

$$\begin{aligned} F_k = \tfrac{1}{4} \sum_{i=0}^{2k-2} [& (-1)^i \langle \Lambda^{2k-i-2}\Phi_1, \Phi_1 \rangle \langle \Lambda^i \Phi_1, \Phi_1 \rangle \\ & + 2\langle \Lambda^{2k-2-i}\Phi_1, \Phi_1 \rangle \langle \Lambda^i \Phi_2, \Phi_2 \rangle + (-1)^i \langle \Lambda^{2k-2-i}\Phi_2, \Phi_2 \rangle \langle \Lambda^i \Phi_2, \Phi_2 \rangle \\ & - \sum_{i=1}^{k-1} \langle \Lambda^{2k-2i-1}\Phi_1, \Phi_2 \rangle \langle \Lambda^{2i-1}\Phi_1, \Phi_2 \rangle + \langle \Lambda^{2k-1}\Phi_1, \Phi_2 \rangle \qquad k \geqslant 1. \end{aligned}$$

If (Φ_1, Φ_2) satisfies both commuting integrable Hamiltonian systems (35) and (36), then ψ given by (34) solves the MKdV_3 equation (19). Similarly, if (Φ_1, Φ_2) is a solution to two commuting finite-dimensional Hamiltonian systems (35) and

$$\Phi_{1t_5} = \frac{\partial H_5}{\partial \Phi_2} \qquad \Phi_{2t_5} = -\frac{\partial H_5}{\partial \Phi_1} \tag{37}$$

with

$$H_5 = 4^2(F_3 + 2F_1F_2 + 2F_1^3)$$

or

$$\Phi_{1t_7} = \frac{\partial H_7}{\partial \Phi_2} \qquad \Phi_{2t_7} = -\frac{\partial H_7}{\partial \Phi_1} \tag{38}$$

with

$$H_7 = 4^3[F_4 + 2F_1F_3 + F_2^2 + 6F_1^2F_2 + 5F_1^4]$$

then ψ given by (34) satisfies the MKdV_5 equation (21) or the MKdV_7 equation (25), respectively. This means that if (Φ_1, Φ_2) satisfies three commuting integral systems (35), (36) and (37) (or (38)), then $u = 2(\langle \Phi_1, \Phi_1 \rangle + \langle \Phi_2, \Phi_2 \rangle)^2$ is a solution to BKP_1 equation (7) (or BKP_2 equation (22)).

It is clear that the above results also provide a way to obtain some kinds of solutions to the BKP equation through solving three commuting finite-dimensional integrable Hamiltonian systems.

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